Math 103 – Intermediate Algebra
Final Exam Review Practice Problems

The final exam covers Chapter 2, Chapter 3, Sections 4.1 – 4.3, Chapter 5, Sections 6.1 - 6.4, 6.6 - 6.7, Chapter 7, Chapter 8, and Chapter 9. The list of materials and skills you should have mastered for the final, as well as a list of review problems, are given on the departmental review sheet. Below are some extra practice problems. They are not meant to be completely comprehensive.

1. Simplify the following expressions. Write the answers with positive exponents only.

a) $\left(\dfrac{3a^{-2}b^{3}c}{6a^{-1}b^{-4}c^{2}}\right)^{-5}$

b) $\dfrac{10x^{2}y}{y^{-3}}\left(\dfrac{4x^{-2}}{15y^{-3}}\right)$

c) $\left(\dfrac{x^{7/3}}{x^{1/3}}\right)^{1/2}$

d) $(-3a^{-1/3})^{-1}(9a^{2/3})$

2. Write the polynomial in standard form. $16 + 4x^3 - x^4 + 3x^2 - 2x$

3. Factor the following polynomials completely.

a) $9x^3 - 39x^2 - 30x$

b) $16x^4 - 81y^4$

c) $8z^3 + 1$

4. Perform the indicated operation and simplify.

a) $(5x^4 - 6x^3 + x - 7) - (-2x^4 + 3x^3 + 4x^2 + 7x - 3)$

b) $(x^2 + 3x - 2)(x - 5)$

c) $(3x - 1)^2$

d) $\dfrac{3}{x+3} - \dfrac{x+1}{x^2+5x+6}$

e) $\dfrac{3}{x} - \dfrac{2}{x+2}$

f) $\dfrac{x^2+4x+4}{x^2-x-6}\cdot\dfrac{2x^2-x-15}{x^2+2x}$

g) $\dfrac{x^2-4}{12x+3}\div\dfrac{3x^2+5x-2}{8x+2}$

h) $4+3\sqrt{2}-2\sqrt{3}-5\sqrt{12}+\sqrt{18}+2\sqrt{25}$

i) $\left(7-\sqrt{3}\right)\left(\sqrt{6}+2\right)$

5. Simplify

a) $\dfrac{2x^2-4x}{x^2-x-2}$

b) $\dfrac{\frac{5}{x}-2}{\frac{x}{3}-\frac{3}{x}}$

c) $\dfrac{\frac{1}{x+2}}{\frac{4}{x^2-4}+1}$

6. Evaluate the expression without a calculator. If the answer is not real, write it in *i*-form.

a) $\sqrt{(-7)^2}$

b) $\sqrt{-7^2}$

c) $\sqrt{\dfrac{49}{121}}$

d) $\sqrt[3]{(-8)^3}$

e) $-\sqrt[3]{-8}$

7. Rewrite the expression using rational exponents and simplify.

a) $y\sqrt{y}$ b) $\dfrac{x^2}{\sqrt[3]{x^2}}$ c) $\left(\sqrt{a}\cdot\sqrt[4]{a^3}\right)^2$

8. Simplify the expressions. Use absolute values when necessary.

a) $\sqrt{50y^2x^5}$ b) $\sqrt[3]{54a^6b^5}$ c) $\sqrt[5]{\dfrac{y^6}{32}}$

9. Rationalize the denominator and simplify.

a) $\sqrt{\dfrac{3}{5x}}$ b) $\dfrac{2\sqrt[3]{3}}{\sqrt[3]{12x}}$ c) $\dfrac{1}{3-\sqrt{7}}$

10. Perform the indicated operations and write the answer in standard form.

a) $\sqrt{-27}-\sqrt{-12}$ b) $(5+3i)(1-i)$

11. Evaluate (if possible) without a calculator:

a.) $\log_7 \dfrac{1}{\sqrt{7}}$ b) $\log_2 64$ c) $\log_3(-15)$

12. Use the properties of logarithms to expand or condense the expression:

a) $3\log_3 x-\dfrac{1}{2}\log_3 y-2\log_3 z$ b) $\ln\left(\dfrac{x}{2z}\right)$

13. Solve the equation.

a) $5(x-7)=3x+2$ b) $\dfrac{x}{2}-1=\dfrac{x}{4}+2$

c) $|4x-3|+2=7$

d) $|2x-7|=-2$

e) $x(2x-9)=5$

f) $2y-y^2=0$

g) $x^3-5x^2-x+5=0$

h) $\frac{2x}{x+3}+\frac{1}{x}=2$

i) $\frac{x}{x+1}+\frac{2}{x-5}=\frac{12}{x^2-4x-5}$

j) $\sqrt{x+7}=x-5$

k) $\sqrt{x+5}-1=\sqrt{x}$

l) $x^{2\backslash 5}-2x^{1/5}-15=0$

m) $(x^2-3)^2-7(x^2-3)+6=0$

n) $3^{x-5}=27$

o) $-7e^{-2x} + 8 = 3$

p) $\log_5(x-3) = 2$

q) $\log_2 x + \log_2(x-2) = 3$

14. Solve by the Square Root Property. Find both real and complex roots.

a) $3(x-3)^2 + 7 = 0$

b) $(2x+1)^2 = 9$

15. Solve both by completing the square and by using the Quadratic Formula. Find both real and complex roots.

a) $x^2 - x - 1 = 0$

b) $2x^2 + 3 = -8x$

16. Solve the systems using the method of substitution. If the system has no solution or infinitely many solutions, say so.

a) $\begin{cases} 2x - 3y = 11 \\ x + 4y = 0 \end{cases}$

b) $\begin{cases} 3x - 2y = 12 \\ 4y - 6x = 12 \end{cases}$

17. Solve the systems using the method of elimination. If the system has no solution or infinitely many solutions, say so.

a) $\begin{cases} 0.3x + 0.5y = 1.4 \\ x + 2y = 1 \end{cases}$.

b) $\begin{cases} 5x + 8y = 14 \\ 4x - 9y = -17 \end{cases}$

18. Solve the system.

$$\begin{cases} x + 2y - z = -3 \\ 3x + 3y = 6 \\ x + 2y - 2z = -5 \end{cases}$$

19. Solve the inequality and sketch the system on the real number line.

a) $3 - 8x \geq 15 - x + 2$

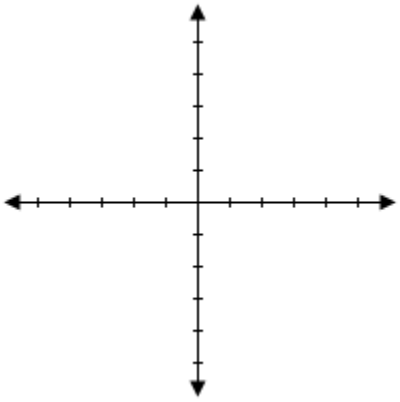
b) $-4 < 1 - 5x \leq 0$

c) $|2x + 3| - 2 \leq 9$

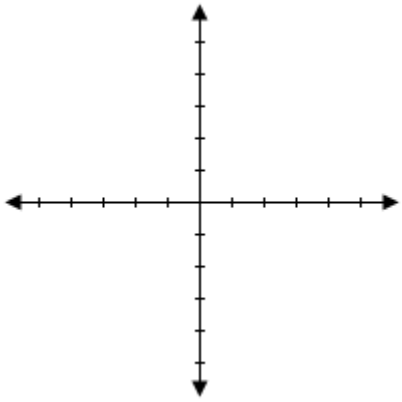
d) $|x + 8| > 1$

20. Sketch the graph of the solution of the linear inequalities.

a) $2x + 3y - 12 > 0$



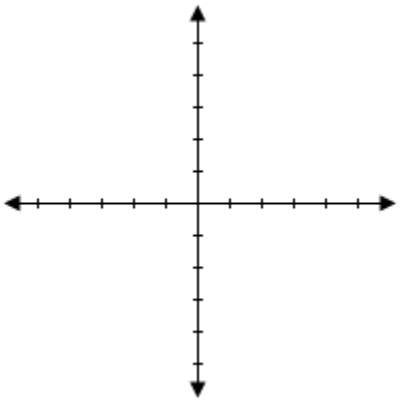
b) $y \leq 4$

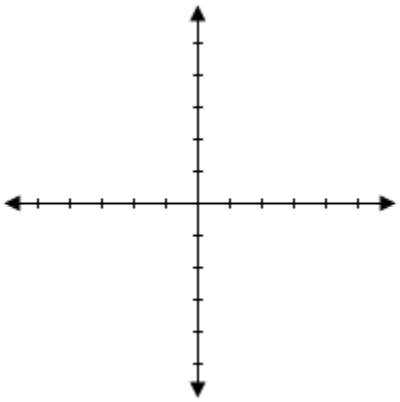


21. Given the following equations (i) find the x and y intercepts (if any); and (ii) sketch a graph of the equation.

a) $y = 4 - x^2$

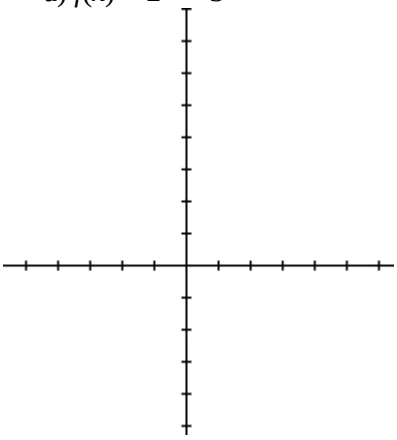
b) $y = \sqrt{x - 2}$



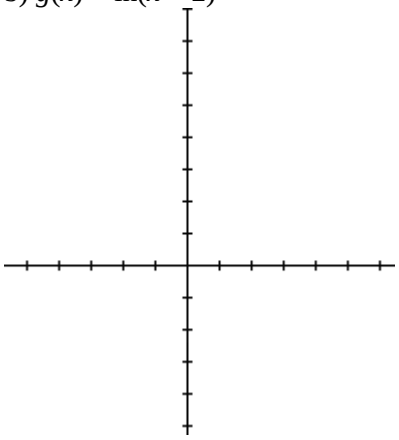


22. Given the following functions (i) find the horizontal or vertical asymptote; and (ii) sketch a graph of the function.

a) $f(x) = 2^{-x} + 3$



b) $g(x) = \ln(x + 2)$



23. $A = \{1, 2, 3, 4\}$ $B = \{5, 6, 7, 8, 9\}$
Determine which set of ordered pairs represents a function from A to B.

a) {(1, 5), (2, 5), (3, 5), (4, 5)}

b) {(1, 5), (2, 6), (1, 7), (3, 8), (4, 9)}

24. Suppose f is a function defined by the ordered pairs { (2, 3), (4, 5), (6, 7), (0, 9) }.

a) What is the domain of f ?

b) What is the range of f ?

25. Evaluate the function as indicated and simplify.

a) $g(x) = -x^2 + 2x - 4$

i) $g(3)$

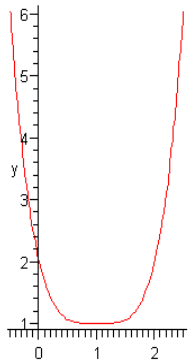
ii) $g(2a) + g(1)$

b) $f(x) = \begin{cases} 2x - 1, & x < 1 \\ x + 5, & x \geq 1 \end{cases}$

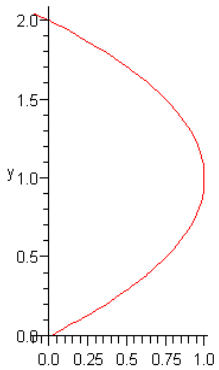
i) $f(0)$ ii) $f(6)$ ii) $f(-1) + f(1)$

26. Use the Vertical Line Test to determine whether y is a function of x.

a) $y - x^4 + 4x^3 - 6x^2 + 4x = 2$



b) $x + y^2 - 2y = 0$



27. Write the equation of the line that passes through the points (1, 7), and (−5, 4) in three different forms: slope-intercept form, point-slope form, and general form.

28. Write the equation of the line in general form that passes through the point (3, 5) and is perpendicular to the line $3x + 6y = 12$.

29. Write the equation of the line in slope-intercept form that passes through the point (−2, 4) and is parallel to the line $x - 2y = 5$.

30. Write the equation of the vertical line that passes through the point (4, −1).

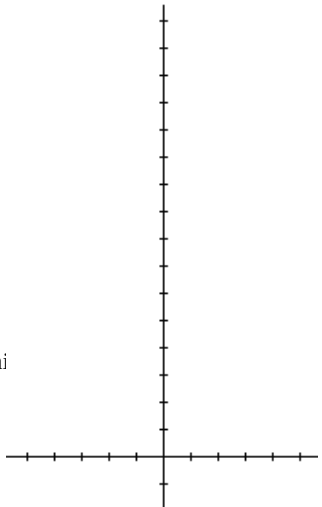
31. Write the equation of the horizontal line that passes through the point (17, 5).

32. Write the equation of the parabola in standard form, and find the vertex of its graph.
$y = 3x^2 - 3x + 1$

33. Let $f(x) = -x^2 + 2x + 15$

a) State whether the graph of $f(x)$ opens upward or downward, and explain how you know.

b) Find the y-intercept of the graph of $f(x)$.



c) Find the x-intercepts of the graph of $f(x)$.

d) Find the vertex of the graph of $f(x)$.

e) Sketch the graph of $f(x)$.

34. Evaluate the function as indicated. Round to 2 decimal places.
a) $f(x) = 1000(1.05)^{2x}$, find $f(4)$.

b) $g(t) = \dfrac{100}{1+e^{-0.01t}}$, find $g(3)$.

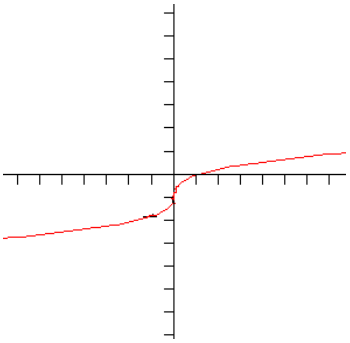
35. Let $f(x) = 2x + 1$, $g(x) = x^2 - 5$. Find the following functions and simplify.
a) $(f \circ g)(x)$ b) $(g \circ f)(x)$

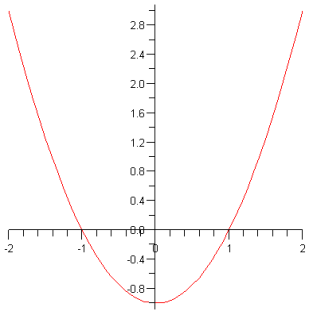
c) $(f \circ f)(x)$ d) $f^{-1}(x)$

36. Verify algebraically that the functions f and g are inverse functions of each other.
$f(x) = \dfrac{3x-2}{4}$, $g(x) = \dfrac{4x+2}{3}$

37. Use a calculator and the change-of-base formula to evaluate the logarithm $\log_{13}7$.

38. Use the horizontal line test to determine if the function is one-to-one. If it is one-to-one, find its inverse and sketch the graph of its inverse on the same set of axes.
a) $f(x) = \sqrt[3]{x} - 1$ b) $f(x) = x^2 - 1$





39. Solve for the specified variable.

a) $A = \frac{bh}{2}$ Solve for h.

b) $I = Prt$ Solve for t.

c) $P = 2l + 2w$ Solve for l.

d) $S = C + Cr$, Solve for C.

40. F varies inversely as the square of m, and $F = 200$ when $m = 2$. Find the constant of proportionality and write an equation that relates the variables. Then find F when $m = 5$.

41. A computer screen has a diagonal length of 17 inches, and is 13 inches wide. How tall is it? Round your answer to the nearest tenth of an inch.

42. A company's profit function P, in thousands of dollars, from selling x units of a product is given by $P(x) = -3600 + 400x - 4x^2$. What level of production maximizes profit, and what is the maximum possible profit?

43. If you give your hair stylist $15 for a haircut that costs $12.50, what is the tip rate?

44. A grocer mixes two kinds of coffee together, cheaper coffee costing 3.50 a pound and a gourmet coffee costing $8.00 a pound to get 30 pounds of a "house blend" that costs 4.25 per pound. How much did he use of each kind?

45. Two cars start at the same location and travel in the opposite direction. One is going 40 miles per hour and the other is going 60 miles per hour. How long before they are 10 miles apart?

46. One little pig can build a house in 4 hours and his brother can build one in 3. How long will it take if they work together?

47. The base of a triangle is 3 inches less than its height. The area of the triangle is 20 square inches. Find the base and height of the triangle.

49. A tennis ball is tossed vertically upward from a height of 4 feet according to the height equation $h = -16t^2 + 8t + 4$, where h is the height of the tennis ball in feet and t is the time in seconds.

    a) After how many seconds is the height 4 feet again?

    b) After how many seconds does the ball hit the ground?

50. A radioactive substance decays according to the formula $A = 10e^{kt}$, where t is the time in years. The half-life of this substance is 70 years. This means if 10 gram is present now, 5 grams will be present in 70 years.

    a) Find the constant k.

b) If 10 grams are present now, how much will be present in 100 years?

51. You have $3000 that you want to invest in a bank account that pays 2.5% annual interest compounded continuously. How long do you need to need to leave it in the account if you want to double the amount of your money, assuming you will make no other deposits or withdrawals.
(The formula for continuous compounding is $A = Pe^{rt}$.)